EXAMPLE 13 Free-Falling Objects. The formula $s = 16t^2$ is used to approximate the distance s, in feet, that an object falls freely from rest in t seconds. The highest point of the Mike O'Callaghan–Pat Tillman Memorial Bridge is 890 ft above the Colorado River. How long will it take a stone to fall from the bridge to the river? Round to the nearest tenth of a second.
Source: www.desertusa.com

SOLUTION

1. **Familiarize.** We agree to disregard air resistance and use the given formula.
2. **Translate.** We substitute into the formula:

$$s = 16t^2$$
$$890 = 16t^2.$$

3. **Carry out.** We solve for t:

$$890 = 16t^2$$
$$55.625 = t^2$$
$$\sqrt{55.625} = t \quad \text{Using the principle of square roots; rejecting the negative square root since } t \text{ cannot be negative in this problem}$$
$$7.5 \approx t. \quad \text{Using a calculator and rounding to the nearest tenth}$$

4. **Check.** Since $16(7.5)^2 = 900 \approx 890$, our answer checks.
5. **State.** It takes about 7.5 sec for a stone to fall freely from the bridge to the river.

Try Exercise 85.

8.1 Exercise Set

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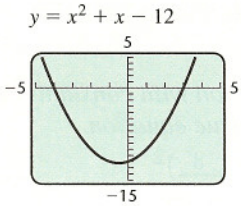
Concept Reinforcement *Complete each of the following to form a true statement.*

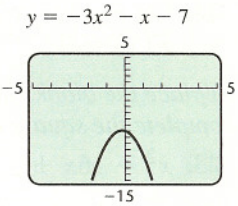
1. The principle of square roots states that if $x^2 = k$, then $x =$ ____ or $x =$ ____. $\sqrt{k}; -\sqrt{k}$
2. If $(x + 5)^2 = 49$, then $x + 5 =$ ____ or $x + 5 =$ ____. $7; -7$
3. If $t^2 + 6t + 9 = 17$, then $(\_\_\_\_)^2 = 17$ and ____ $= \pm\sqrt{17}$. $t + 3; t + 3$
4. The equations $x^2 + 8x +$ ____ $= 23$ and $x^2 + 8x = 7$ are equivalent. 16
5. The expressions $t^2 + 10t +$ ____ and $(t +$ ____$)^2$ are equivalent. $25; 5$
6. The expressions $x^2 - 6x +$ ____ and $(x -$ ____$)^2$ are equivalent. $9; 3$

Determine the number of real-number solutions of each equation from the given graph.

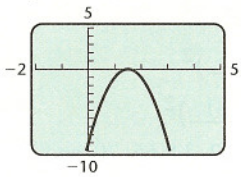
7. $x^2 + x - 12 = 0$ 2



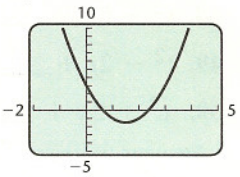
8. $-3x^2 - x - 7 = 0$ 0



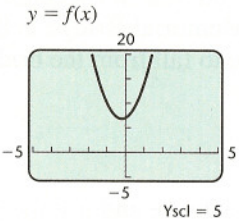
9. $4x^2 + 9 = 12x$ 1



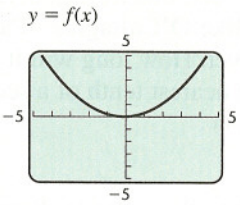
10. $2x^2 + 3 = 6x$ 2



11. $f(x) = 0$ 0



12. $f(x) = 0$ 1



Solve.

13. $x^2 = 100$ ± 10

14. $t^2 = 144$ ± 12

15. $p^2 - 50 = 0$ $\pm 5\sqrt{2}$

16. $c^2 - 8 = 0$ $\pm 2\sqrt{2}$

17. $4x^2 = 20$ $\pm\sqrt{5}$

18. $7x^2 = 21$ $\pm\sqrt{3}$

19. $x^2 = -4$ $\pm 2i$

20. $x^2 = -9$ $\pm 3i$

21. $9x^2 - 16 = 0$ $\pm\frac{4}{3}$

22. $25x^2 - 4 = 0$ $\pm\frac{2}{5}$

23. $5t^2 - 3 = 4$ ⊡

24. $3t^2 - 1 = 6$ ⊡

25. $4d^2 + 81 = 0$ $\pm\frac{9}{2}i$

26. $25y^2 + 16 = 0$ $\pm\frac{4}{5}i$

27. $(x - 1)^2 = 49$ $-6, 8$

28. $(x + 2)^2 = 25$ $-7, 3$

29. $(a - 13)^2 = 18$ $13 \pm 3\sqrt{2}$

30. $(a + 5)^2 = 8$ ⊡

31. $(x + 1)^2 = -9$ $-1 \pm 3i$

32. $(x - 1)^2 = -49$ ⊡

33. $\left(y + \frac{3}{4}\right)^2 = \frac{17}{16}$ ⊡

34. $\left(t + \frac{3}{2}\right)^2 = \frac{7}{2}$ ⊡

35. $x^2 - 10x + 25 = 64$ $-3, 13$

36. $x^2 - 6x + 9 = 100$ $-7, 13$

37. Let $f(x) = x^2$. Find x such that $f(x) = 19$. $\pm\sqrt{19}$

38. Let $f(x) = x^2$. Find x such that $f(x) = 11$. $\pm\sqrt{11}$

39. Let $f(x) = (x - 5)^2$. Find x such that $f(x) = 16$. $1, 9$

40. Let $g(x) = (x - 2)^2$. Find x such that $g(x) = 25$. $-3, 7$

41. Let $F(t) = (t + 4)^2$. Find t such that $F(t) = 13$. $-4 \pm \sqrt{13}$

42. Let $f(t) = (t + 6)^2$. Find t such that $f(t) = 15$. $-6 \pm \sqrt{15}$

Aha!

43. Let $g(x) = x^2 + 14x + 49$. Find x such that $g(x) = 49$. $-14, 0$

44. Let $F(x) = x^2 + 8x + 16$. Find x such that $F(x) = 9$. $-7, -1$

Replace the blanks in each equation with constants to complete the square and form a true equation.

45. $x^2 + 16x + \underline{64} = (x + \underline{8})^2$

46. $x^2 + 8x + \underline{16} = (x + \underline{4})^2$

47. $t^2 - 10t + \underline{25} = (t - \underline{5})^2$

48. $t^2 - 6t + \underline{9} = (t - \underline{3})^2$

49. $t^2 - 2t + \underline{1} = (t - \underline{1})^2$

50. $x^2 + 2x + \underline{1} = (x + \underline{1})^2$

51. $x^2 + 3x + \underline{\frac{9}{4}} = (x + \underline{\frac{3}{2}})^2$

52. $t^2 - 9t + \underline{\frac{81}{4}} = (t - \underline{\frac{9}{2}})^2$

53. $x^2 + \frac{2}{5}x + \underline{\frac{1}{25}} = (x + \underline{\frac{1}{5}})^2$

54. $x^2 + \frac{2}{3}x + \underline{\frac{1}{9}} = (x + \underline{\frac{1}{3}})^2$

55. $t^2 - \frac{5}{6}t + \underline{\frac{25}{144}} = (t - \underline{\frac{5}{12}})^2$

56. $t^2 - \frac{5}{3}t + \underline{\frac{25}{36}} = (t - \underline{\frac{5}{6}})^2$

Solve by completing the square. Show your work.

57. $x^2 + 6x = 7$ $-7, 1$

58. $x^2 + 8x = 9$ $-9, 1$

59. $t^2 - 10t = -23$ $5 \pm \sqrt{2}$

60. $t^2 - 4t = -1$ $2 \pm \sqrt{3}$

61. $x^2 + 12x + 32 = 0$ $-8, -4$

62. $x^2 + 16x + 15 = 0$ $-15, -1$

63. $t^2 + 8t - 3 = 0$ $-4 \pm \sqrt{19}$

64. $t^2 + 6t - 5 = 0$ $-3 \pm \sqrt{14}$

Complete the square to find the x-intercepts of each function given by the equation listed.

65. $f(x) = x^2 + 6x + 7$ ⊡

66. $f(x) = x^2 + 10x - 2$ ⊡

67. $g(x) = x^2 + 9x - 25$ ⊡

68. $g(x) = x^2 + 5x + 2$ ⊡

69. $f(x) = x^2 - 10x - 22$ ⊡

70. $f(x) = x^2 - 8x - 10$ ⊡

Solve by completing the square. Remember to first divide, as in Example 11, to make sure that the coefficient of x^2 is 1.

71. $9x^2 + 18x = -8$ $-\frac{4}{3}, -\frac{2}{3}$

72. $4x^2 + 8x = -3$ $-\frac{3}{2}, -\frac{1}{2}$

73. $3x^2 - 5x - 2 = 0$ $-\frac{1}{3}, 2$

74. $2x^2 - 5x - 3 = 0$ $-\frac{1}{2}, 3$

75. $5x^2 + 4x - 3 = 0$ ⊡

76. $4x^2 + 3x - 5 = 0$ ⊡

77. Find the x-intercepts of the function given by $f(x) = 4x^2 + 2x - 3$. ⊡

78. Find the x-intercepts of the function given by $f(x) = 3x^2 + x - 5$. ⊡

79. Find the x-intercepts of the function given by $g(x) = 2x^2 - 3x - 1$. ⊡

80. Find the x-intercepts of the function given by $g(x) = 3x^2 - 5x - 1$. ⊡

Interest. Use $A = P(1 + r)^t$ to find the interest rate in Exercises 81–84. Refer to Example 12.

81. \$2000 grows to \$2420 in 2 years 10%

82. \$1000 grows to \$1440 in 2 years 20%

83. \$6250 grows to \$6760 in 2 years 4%

84. \$6250 grows to \$7290 in 2 years 8%

⊡ Answers to Exercises 23, 24, 30, 32–34, 65–70, and 75–80 are on p. IA-17.

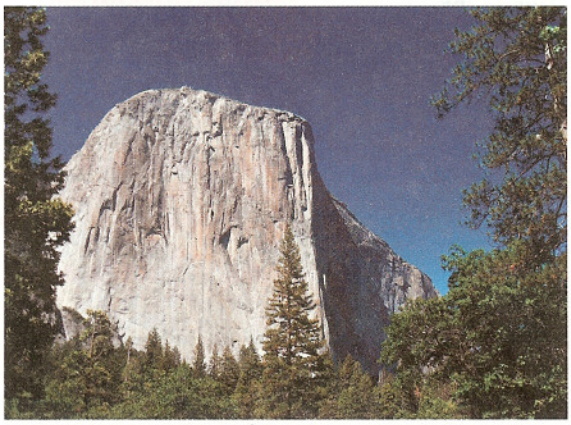
Free-Falling Objects. *Use $s = 16t^2$ for Exercises 85–88. Refer to Example 13 and neglect air resistance.*

85. The Grand Canyon skywalk is 4000 ft above the Colorado River. How long will it take a stone to fall from the skywalk to the river? About 15.8 sec
Source: www.grandcanyonskywalk.com

86. The Sears Tower in Chicago is 1454 ft tall. How long would it take an object to fall freely from the top? About 9.5 sec

87. At 2063 ft, the KVLY-TV tower in North Dakota is the tallest supported tower in the United States. How long would it take an object to fall freely from the top?
Source: North Dakota Tourism Division About 11.4 sec

88. El Capitan in Yosemite National Park is 3593 ft high. How long would it take a carabiner to fall freely from the top? About 15.0 sec
Source: *Guinness World Records* 2008



TW **89.** Explain in your own words a sequence of steps that can be used to solve any quadratic equation in the quickest way.

TW **90.** Describe how to write a quadratic equation that can be solved algebraically but not graphically.

SKILL REVIEW

To prepare for Section 8.2, review evaluating expressions and simplifying radical expressions (Sections 1.2, 7.3, and 7.8).

Evaluate. [1.2]

91. $b^2 - 4ac$, for $a = 3$, $b = 2$, and $c = -5$ 64

92. $b^2 - 4ac$, for $a = 1$, $b = -1$, and $c = 4$ −15

Simplify. [7.3], [7.8]

93. $\sqrt{200}$ $10\sqrt{2}$

94. $\sqrt{96}$ $4\sqrt{6}$

95. $\sqrt{-4}$ $2i$

96. $\sqrt{-25}$ $5i$

97. $\sqrt{-8}$ $2i\sqrt{2}$, or $2\sqrt{2}i$

98. $\sqrt{-24}$ $2i\sqrt{6}$, or $2\sqrt{6}i$

SYNTHESIS

TW **99.** What would be better: to receive 3% interest every 6 months or to receive 6% interest every 12 months? Why?

TW **100.** Example 12 was solved with a graphing calculator by graphing each side of

$$4410 = 4000(1 + r)^2.$$

How could you determine, from a reading of the problem, a suitable viewing window?

Find b such that each trinomial is a square. ±14

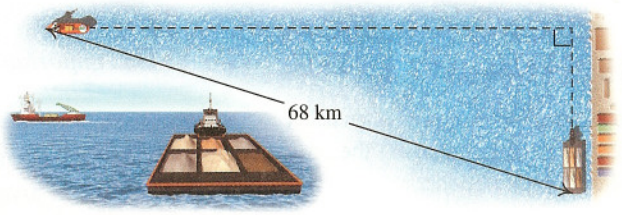
101. $x^2 + bx + 81$ ±18

102. $x^2 + bx + 49$

103. If $f(x) = 2x^5 - 9x^4 - 66x^3 + 45x^2 + 280x$ and $x^2 - 5$ is a factor of $f(x)$, find all a for which $f(a) = 0$. $-\frac{7}{2}, -\sqrt{5}, 0, \sqrt{5}, 8$

104. If $f(x) = \left(x - \frac{1}{3}\right)(x^2 + 6)$ and $g(x) = \left(x - \frac{1}{3}\right)\left(x^2 - \frac{2}{3}\right)$, find all a for which $(f + g)(a) = 0$. $\frac{1}{3}, \pm\frac{2\sqrt{6}}{3}i$

105. *Boating.* A barge and a fishing boat leave a dock at the same time, traveling at a right angle to each other. The barge travels 7 km/h slower than the fishing boat. After 4 hr, the boats are 68 km apart. Find the speed of each boat. Barge: 8 km/h; fishing boat: 15 km/h



106. Find three consecutive integers such that the square of the first plus the product of the other two is 67. 5, 6, 7

Try Exercise Answers: Section 8.1

13. ±10 **17.** $\pm\sqrt{5}$ **23.** $\pm\sqrt{\frac{7}{5}}$, or $\pm\frac{\sqrt{35}}{5}$ **25.** $\pm\frac{9}{2}i$
35. −3, 13 **41.** $-4 \pm \sqrt{13}$ **45.** $x^2 + 16x + 64 = (x + 8)^2$
57. −7, 1 **65.** $(-3 - \sqrt{2}, 0), (-3 + \sqrt{2}, 0)$ **71.** $-\frac{4}{3}, -\frac{2}{3}$
81. 10% **85.** About 15.8 sec